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The solution to Slavnov-Taylor identities in D4 $N = 1$ SYM

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Abstract: D4 $N = 1$ SYM with an arbitrary chiral background superfield as the gauge coupling is considered. The solution to Slavnov-Taylor identities has been given. It has been shown that the solution is unique and allows us to restrict the gauge part of the effective action. Under the effective action in this paper we mean the 1PI diagram generator.

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One of the ways to break supersymmetry is to introduce interactions with background superfields that are space-time independent into the supersymmetric theory. If we do not include these background superfields in the supersymmetry transformation at the component level we have breaking of supersymmetry. This is so-called softly broken supersymmetry. The relation between the theory with softly broken supersymmetry and its rigid counterpart has been studied in refs. ([1]–[6]). The investigation has been performed for singular parts of the effective actions of softly broken and rigid theories. Since the only modification of the classical action from the rigid case to the softly broken case is a replacement of coupling constants of the rigid theory with background superfields, the relation is simple and can be reduced to substitutions of these superfields into renormalization constants of the rigid theory instead of the rigid theory couplings [4, 5]. Later, a relation between full correlators of softly broken and unbroken SUSY quantum mechanics has been found [7]. Non-perturbative results for the terms of the effective action which correspond to the case when chiral derivatives do not act on background superfields have been derived in ref. [8].

The renormalization of the soft theory has been made on the basis of supergraph technique in the ref. [5], and at the level of Slavnov-Taylor identities the renormalization procedure for these theories has been performed more recently in [9] for the case of an arbitrary chiral superfield as the gauge coupling. However, the renormalization procedure deals with infinities that must be removed from the theory. Actually, Slavnov-Taylor (ST) identities contain much more information since they restrict the total effective action Γ. They could be used to extract information about finite parts of proper correlators. This information can be useful in order to calculate process amplitudes in the Minimal Supersymmetric Standard Model within superfield formalism. In this paper we propose the way to find the solution to ST identities and show that the solution is unique.

The notation used for the D4 supersymmetry and for the classical action S of the theory with softly broken supersymmetry are the same as those have been used in our previous paper [9]. In this work we concentrate on the pure gauge supersymmetric theory.

The gauge fixing condition [9] is taken as

$$D^2\frac{V(x,\theta,\bar\theta)}{\sqrt{\tilde\alpha}} = \bar f(\bar y,\bar\theta)\,, \qquad \bar D^2\frac{V(x,\theta,\bar\theta)}{\sqrt{\tilde\alpha}} = f(y,\theta)\,, \tag{1}$$

with an arbitrary background gauge fixing superfield $\tilde\alpha$. We have shown in [9] that this modification of the gauge fixing condition is necessary and important for the renormalization procedure in the softly broken SYM.

With the modification (1) the total gauge part of the classical action is

$$S_{\text{gauge}} = \int d^4y d^2\theta\; S\frac{1}{2^7}\,\mathrm{Tr}\;W_\alpha W^\alpha + \int d^4\bar y d^2\bar\theta\;\bar S\frac{1}{2^7}\,\mathrm{Tr}\;\bar W^{\dot\alpha}\bar W_{\dot\alpha} +$$

$$+\int d^4x d^2\theta d^2\bar\theta\,\frac{1}{16}\,\mathrm{Tr}\,\left(\bar D^2\frac{V}{\sqrt{\tilde\alpha}}\right)\left(D^2\frac{V}{\sqrt{\tilde\alpha}}\right)+$$
$$+\int d^4y d^2\theta\,\frac{i}{2}\,\mathrm{Tr}\;b\;\bar D^2\left(\frac{\delta_{\bar c,c}V}{\sqrt{\tilde\alpha}}\right)+\int d^4\bar y d^2\bar\theta\,\frac{i}{2}\,\mathrm{Tr}\;\bar b\;D^2\left(\frac{\delta_{\bar c,c}V}{\sqrt{\tilde\alpha}}\right). \quad (2)$$

Here S is an arbitrary chiral superfield (in general, x-dependent). For example, in case of the softly broken supersymmetry we have

$$S=\frac{1}{g^2}\left(1-2m_A\theta^2\right).$$

In the softly broken case the path integral describing the quantum soft theory is defined as

$$Z[J,\eta,\bar\eta,\rho,\bar\rho,K,L,\bar L]=\int dV\;dc\;d\bar c\;db\;d\bar b\;\times$$
$$\times\;\exp i\Big[S_{\rm gauge}+2\;\;\mathrm{Tr}(JV+i\eta c+i\bar\eta\bar c+i\rho b+i\bar\rho\bar b)+$$
$$+\;2\;\;\mathrm{Tr}\left(iK\delta_{\bar c,c}V+Lc^2+\bar L\bar c^2\right)\Big]. \quad (3)$$

The third term in the brackets is BRST invariant since the external superfields K and L are BRST invariant by definition. All fields in the path integral are in the adjoint representation of the gauge group. For the sake of brevity we omit the symbol of integration in the terms with external sources, keeping in mind that it is the full superspace measure for vector superfields and the chiral measure for chiral superfields.

Having shifted the antighost superfields b and $\bar b$ by arbitrary chiral and antichiral superfields, respectively, or, having made the change of variables in the path integral (3) which are the BRST transformations of the total gauge action (2), we get ghost equations [10]

$$\frac{\delta\Gamma}{\delta\bar b}-\frac14 D^2\frac{1}{\sqrt{\tilde\alpha}}\frac{\delta\Gamma}{\delta K}=0\,,\qquad \frac{\delta\Gamma}{\delta b}-\frac14\bar D^2\frac{1}{\sqrt{\tilde\alpha}}\frac{\delta\Gamma}{\delta K}=0 \quad (4)$$

and ST identities

$$\mathrm{Tr}\left[\frac{\delta\Gamma}{\delta V}\frac{\delta\Gamma}{\delta K}-i\frac{\delta\Gamma}{\delta c}\frac{\delta\Gamma}{\delta L}+i\frac{\delta\Gamma}{\delta\bar c}\frac{\delta\Gamma}{\delta\bar L}-\frac{\delta\Gamma}{\delta b}\left(\frac{1}{32}\bar D^2D^2\frac{V}{\sqrt{\tilde\alpha}}\right)-\right.$$
$$\left.-\frac{\delta\Gamma}{\delta\bar b}\left(\frac{1}{32}D^2\bar D^2\frac{V}{\sqrt{\tilde\alpha}}\right)\right]=0\,, \quad (5)$$

respectively. In the ref. [9] it has been shown that the ghost equations (4) and the ST identities (5) restrict the singular part of the effective action of the SYM theory with softly broken supersymmetry which corresponds to the theory with classical action (2) to

$$\begin{aligned}\Gamma_{\text{sing}} = &\int d^4y d^2\theta\, S\frac{1}{2^7}\,\tilde{z}_S\,\mathrm{Tr}\;W_\alpha\left(\frac{V}{\tilde{z}_1}\right)W^\alpha\left(\frac{V}{\tilde{z}_1}\right)+\text{H.c.}+\\ &+\int d^4xd^2\theta d^2\bar{\theta}\;\mathrm{Tr}\,\frac{1}{32}\frac{V}{\sqrt{\tilde{\alpha}}}\left(D^2\bar{D}^2+\bar{D}^2D^2\right)\frac{V}{\sqrt{\tilde{\alpha}}}+\\ &+\int d^4xd^2\theta d^2\bar{\theta}\;2i\;\mathrm{Tr}\left((b+\bar{b})\frac{\tilde{z}_1}{\sqrt{\tilde{\alpha}}}+K\tilde{z}_1\right)\;\left[\delta_{\bar{c},c}\left(\frac{V}{\tilde{z}_1}\right)\right],\end{aligned} \tag{6}$$

where the singular superfields $\tilde{z}_1$ and $\tilde{z}_S$ are related to the renormalization constants of the rigid theory in a simple way [9],

$$\begin{aligned}\tilde{z}_1\left(S,\bar{S},\sqrt{\tilde{\alpha}}\right) &= z_1\left(g^2\to\left(\frac{S+\bar{S}}{2}\right)^{-1},\sqrt{\alpha}\to\sqrt{\tilde{\alpha}}\right),\\ \tilde{z}_S(S) &= z_{g^2}\left(\frac{1}{g^2}\to S\right).\end{aligned}$$

Here we show that the total effective action can be restricted in an analogous way. We go alone the line of our previous paper [9]. The ghost equations (4) restrict the dependence of Γ on the antighost superfields and on the external source K to an arbitrary dependence on their combination

$$\left(b+\bar{b}\right)\frac{1}{\sqrt{\tilde{\alpha}}}+K\,. \tag{7}$$

We can present this dependence of the effective action on the external source K in terms of a series

$$\Gamma=\mathcal{F}_0+\sum_{n=1}\int d^8z_1d^8z_2\cdots d^8z_n\mathcal{F}_n\left(z_1,z_2,\ldots,z_n\right)K(z_1)K(z_2)\cdots K(z_n)\,. \tag{8}$$

According to (7) we should write $(b(z_i)+\bar{b}(z_i))/\sqrt{\tilde{\alpha}(z_i)}+K(z_i)$ instead of $K(z_i)$, but we do not do it for the sake of brevity. The coefficient functions of this expansion are in their turn functionals of other superfields,

$$\mathcal{F}_n=\mathcal{F}_n\left[V,c,\bar{c},L,\bar{L}\right],$$

whose coefficient functions are ghost-antighost-vector correlators. $\mathcal{F}_0$ is a K-independent part of the effective action.

Our purpose is to restrict the expansion (8) by using ST identities (5). At the moment for simplicity we suggest that the coefficient functions $\mathcal{F}_n$ for $n>1$ do not depend on the external sources $L,\bar{L}$ (in what follows we argue this conjecture). The total degree of the ghost superfields c and $\bar{c}$ in $\mathcal{F}_n$ must be equal to n since each proper supergraph contains equal number of ghost and antighost superfields among its external legs.

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To start we consider $\mathcal{F}_1(z_1)$ coefficient function in the expansion (8). The corresponding term in (8) is

$$\int d^8z\, d^8z'\; 2i\;\; \mathrm{Tr}\left(\left(b(z)+\bar{b}(z)\right) \frac{1}{\sqrt{\tilde{\alpha}(z)}} + K(z) \right)\; G^{(2)}_{[S,\bar{S},\sqrt{\tilde{\alpha}}]}(z-z') \times$$
$$\times \left(c(z')+\bar{c}(z')\right), \tag{9}$$

where $G^{(2)}_{[S,\bar{S},\sqrt{\tilde{\alpha}}]}(z-z')$ is a 2-point ghost proper correlator. It is functional of external background superfields $S, \bar{S}, \sqrt{\tilde{\alpha}}$. It is a hermitian kernel of the above integral,

$$\left(G^{(2)}_{[S,\bar{S},\sqrt{\tilde{\alpha}}]}\right)^{\dagger} = G^{(2)}_{[S,\bar{S},\sqrt{\tilde{\alpha}}]}\,.$$

We make the change of variables in the effective active action Γ,

$$\begin{aligned} K(z) &= \int d^8z'\;\tilde{K}(z')\;{}^{(-1)}G^{(2)}_{[S,\bar{S},\sqrt{\tilde{\alpha}}]}(z-z')\,,\\ V(z) &= \int d^8z'\;\tilde{V}(z')\;G^{(2)}_{[S,\bar{S},\sqrt{\tilde{\alpha}}]}(z-z')\,, \end{aligned} \tag{10}$$

where ${}^{(-1)}G^{(2)}_{[S,\bar{S},\sqrt{\tilde{\alpha}}]}(z-z')$ is a 2-point connected ghost correlator,

$$\int d^8z'\;G^{(2)}_{[S,\bar{S},\sqrt{\tilde{\alpha}}]}(z_1-z')\;{}^{(-1)}G^{(2)}_{[S,\bar{S},\sqrt{\tilde{\alpha}}]}(z_2-z') = \delta^{(8)}(z_2-z_1)\,.$$

In terms of new variables, the effective action

$$\begin{aligned} \Gamma\left[V,c,\bar{c},b,\bar{b},K,L,\bar{L}\right] &= \Gamma\left[V(\tilde{V}),c,\bar{c},b,\bar{b},K(\tilde{K}),L,\bar{L}\right]\\ &\equiv \tilde{\Gamma}\left[\tilde{V},c,\bar{c},b,\bar{b},\tilde{K},L,\bar{L}\right], \end{aligned} \tag{11}$$

satisfies to modified ST identities

$$\mathrm{Tr}\left[\frac{\delta\tilde{\Gamma}}{\delta\tilde{V}}\frac{\delta\tilde{\Gamma}}{\delta\tilde{K}} - i\frac{\delta\tilde{\Gamma}}{\delta c}\frac{\delta\tilde{\Gamma}}{\delta L} + i\frac{\delta\tilde{\Gamma}}{\delta\bar{c}}\frac{\delta\tilde{\Gamma}}{\delta\bar{L}} - \frac{\delta\tilde{\Gamma}}{\delta b}\left(\frac{1}{32}\bar{D}^2D^2\frac{\tilde{V}\star G^{(2)}_{[S,\bar{S},\sqrt{\tilde{\alpha}}]}}{\sqrt{\tilde{\alpha}}}\right) - \right.$$
$$\left. - \frac{\delta\tilde{\Gamma}}{\delta\bar{b}}\left(\frac{1}{32}D^2\bar{D}^2\frac{\tilde{V}\star G^{(2)}_{[S,\bar{S},\sqrt{\tilde{\alpha}}]}}{\sqrt{\tilde{\alpha}}}\right)\right] = 0\,, \tag{12}$$

where a new notation is introduced for the convolutions (10)

$$K = \tilde{K}\star{}^{(-1)}G^{(2)}_{[S,\bar{S},\sqrt{\tilde{\alpha}}]}\,,\qquad V = \tilde{V}\star\;G^{(2)}_{[S,\bar{S},\sqrt{\tilde{\alpha}}]}\,.$$

Having made the change (11), we can present (9) as

$$\int d^8z\,d^8z'\;2i\;\;\mathrm{Tr}\left(b(z)+\bar{b}(z)\right)\frac{1}{\sqrt{\tilde{\alpha}(z)}}\;G^{(2)}_{[S,\bar{S},\sqrt{\tilde{\alpha}}]}(z-z')\left(c(z')+\bar{c}(z')\right) +$$
$$+\int d^8z\;2i\;\;\mathrm{Tr}\;\tilde{K}(z)\;\left(c(z)+\bar{c}(z)\right).$$

Hence, the first term in the ST identities (12) restricts $\tilde{V}^2$ term in $\tilde{K}$-independent part of $\tilde{\Gamma}$ to the form

$$\int d^8z\ f\left[S,\bar{S},\sqrt{\tilde{\alpha}}\right]\ \left(D_\alpha\tilde{V}\right)\left(\bar{D}^2D^\alpha\tilde{V}\right)+\text{H.c.}\,,$$

where $f\left[S,\bar{S},\sqrt{\tilde{\alpha}}\right]$ is a chiral functional of the external background superfields. We work in terms of the perturbation theory. Hence, we can apply the no-renormalization theorem that states the absence of perturbative corrections to the superpotential. This means that the superpotential of (3)

$$\int d^4yd^2\theta\ 2\,\text{Tr}\ Lc^2+\int d^4\bar{y}d^2\bar{\theta}\ 2\,\text{Tr}\ \bar{L}\bar{c}^2$$

remains unchanged in the effective action. The first term in ST identities also can be expanded in terms of $\tilde{K}$,

$$\frac{\delta\tilde{\Gamma}}{\delta\tilde{V}}\frac{\delta\tilde{\Gamma}}{\delta\tilde{K}}=\mathcal{M}_0+\sum_{n=1}\int d^8z_1d^8z_2\cdots d^8z_n\ \mathcal{M}_n\left(z_1,z_2,\ldots,z_n\right)\tilde{K}(z_1)\tilde{K}(z_2)\cdots\tilde{K}(z_n)\,,$$

where $\mathcal{M}_0$ is $\tilde{K}$-independent part of $\frac{\delta\tilde{\Gamma}}{\delta\tilde{V}}\frac{\delta\tilde{\Gamma}}{\delta\tilde{K}}$. We can consider it as a result of infinitesimal transformation $\tilde{\Gamma}$ in which instead of $\tilde{V}$ we have substituted

$$\tilde{V}\longrightarrow\tilde{V}+\frac{\delta\tilde{\Gamma}}{\delta\tilde{K}}\,. \tag{13}$$

Hence, the term of the order $\tilde{V}$ in $\mathcal{F}_1$ is fixed completely by the $\tilde{V}^0$ term in $\mathcal{F}_1$ since the only contribution into the $\tilde{V}^0$ term in $\mathcal{M}_1$ comes from this $\tilde{V}$ term in $\mathcal{F}_1$. Indeed, another possible contribution into the $\tilde{V}^0$ term of $\mathcal{M}_1$ could go from $\mathcal{F}_2$ substituted in accordance with (13) into $\mathcal{F}_0$ but it is with necessity of the order of $\tilde{V}$ at least. Hence, the term of the order of $\tilde{V}$ in $\mathcal{F}_1$ is the term of the order of $\tilde{V}$ in $\delta_{\bar{c},c}\tilde{V}$. All the terms in $\mathcal{F}_1$ and $\mathcal{F}_0$ of higher orders in $\tilde{V}$ are fixed by themselves in an iterative way. The only solution to $\mathcal{F}_1$ is $\delta_{\bar{c},c}\tilde{V}$. Starting from the fourth degree of $\tilde{V}$ higher order BRST invariant contributions like

$$\int d^4yd^2\theta\ f_2\left[S,\bar{S},\sqrt{\tilde{\alpha}}\right]\ \text{Tr}\ W_\alpha(\tilde{V})W^\alpha(\tilde{V})W_\beta(\tilde{V})W^\beta(\tilde{V})$$

into $\mathcal{F}_0$ are allowed. Here $f_2\left[S,\bar{S},\sqrt{\tilde{\alpha}}\right]$ is a chiral functional of external background superfields. The following notation is used for brevity

$$W^\alpha\left(V\right)\equiv\bar{D}^2\left(e^{-V}D^\alpha e^V\right).$$

All coefficient functions $\mathcal{F}_n$ with $n>1$ are equal to zero. Indeed, $\mathcal{F}_2$ contributes into $\mathcal{M}_1$ but we do not have anything that can compensate this contribution by ghost transformations induced by the second and third terms in the modified ST

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identities (12). Hence, $\mathcal{F}_2 = 0$. If we consider $\mathcal{F}_3$ it contributes into $\mathcal{M}_2$ and, in general, could be compensated by ghost transformations in $\mathcal{F}_2$. But $\mathcal{F}_2$ is zero, hence, $\mathcal{F}_3$ is also zero. We can repeat the former argument for all higher numbers n of $\mathcal{F}_n$.

It can be also shown that there is no room for the dependence of $\mathcal{F}_n$ on the external superfields L and $\bar{L}$. Indeed, the first degree of $(L+\bar{L})$ could be written in the $\mathcal{F}_1$ term, for example. The corresponding contribution into $\mathcal{M}_1$ due to substitution (13) would be proportional to $(L+\bar{L})$ while a contribution due to variations of the ghost fields in $\mathcal{F}_1$ which are proportional to $(L+\bar{L})$ goes into $\mathcal{M}_2$, and, hence, they can not compensate each other. Hence, the first degree is equal to zero. The same is true for all degrees of $(L+\bar{L})$.

Hence, the effective action can be presented as

$$\begin{aligned}
\tilde{\Gamma} = &\int d^4y d^2\theta \; f\left[S,\bar{S},\sqrt{\tilde{\alpha}}\right] \;\; \mathrm{Tr}\;\; W_\alpha(\tilde{V}) W^\alpha(\tilde{V}) + \mathrm{H.c.} + \\
&+ \int d^4y d^2\theta \; f_2\left[S,\bar{S},\sqrt{\tilde{\alpha}}\right] \;\; \mathrm{Tr}\;\; W_\alpha(\tilde{V}) W^\alpha(\tilde{V}) W_\beta(\tilde{V}) W^\beta(\tilde{V}) + \mathrm{H.c.} + \cdots + \\
&+ \int d^8z \; \frac{1}{32}\,\mathrm{Tr}\, \frac{\tilde{V} \star G^{(2)}_{[S,\bar{S},\sqrt{\tilde{\alpha}}]}}{\sqrt{\tilde{\alpha}}} \left(D^2\bar{D}^2 + \bar{D}^2 D^2\right) \frac{\tilde{V} \star G^{(2)}_{[S,\bar{S},\sqrt{\tilde{\alpha}}]}}{\sqrt{\tilde{\alpha}}} + \\
&+ \int d^8z\; d^8z' \; 2i \;\; \mathrm{Tr}\left(b(z) + \bar{b}(z)\right) \frac{1}{\sqrt{\tilde{\alpha}}}\; G^{(2)}_{[S,\bar{S},\sqrt{\tilde{\alpha}}]}(z-z') \;\, \delta_{\bar{c},c}\tilde{V}(z') + \\
&+ \int d^8z \; 2i \;\; \mathrm{Tr}\;\; \tilde{K}(z)\; \delta_{\bar{c},c}\tilde{V}(z) + \int d^4y d^2\theta \; 2\,\mathrm{Tr}\;\; Lc^2 + \int d^4\bar{y} d^2\bar{\theta} \; 2\,\mathrm{Tr}\;\; \bar{L}\bar{c}^2 .
\end{aligned} \tag{14}$$

where $\cdots$ denote the BRST (gauge) invariant terms of higher orders in $W_\alpha(\tilde{V})$. Now we should go back to the initial variables V and K, that is, we should made the change of variables in $\tilde{\Gamma}$ reversed to (11).

$$\begin{aligned}
\Gamma = &\int d^4y d^2\theta \; f\left[S,\bar{S},\sqrt{\tilde{\alpha}}\right] \;\; \mathrm{Tr}\;\; W_\alpha\left(V \star {}^{(-1)}G^{(2)}_{[S,\bar{S},\sqrt{\tilde{\alpha}}]}\right) W^\alpha\left(V \star {}^{(-1)}G^{(2)}_{[S,\bar{S},\sqrt{\tilde{\alpha}}]}\right) + \\
&+ \mathrm{H.c.} + \int d^4y d^2\theta \; f_2\left[S,\bar{S},\sqrt{\tilde{\alpha}}\right] \;\; \mathrm{Tr}\;\; \left(W_\alpha\left(V \star {}^{(-1)}G^{(2)}_{[S,\bar{S},\sqrt{\tilde{\alpha}}]}\right) \times \right.\\
&\left. \times W^\alpha\left(V \star {}^{(-1)}G^{(2)}_{[S,\bar{S},\sqrt{\tilde{\alpha}}]}\right) W_\beta\left(V \star {}^{(-1)}G^{(2)}_{[S,\bar{S},\sqrt{\tilde{\alpha}}]}\right) W^\beta\left(V \star {}^{(-1)}G^{(2)}_{[S,\bar{S},\sqrt{\tilde{\alpha}}]}\right)\right) + \\
&+ \mathrm{H.c.} + \cdots + \int d^8z \; \frac{1}{32}\,\mathrm{Tr}\, \frac{V}{\sqrt{\tilde{\alpha}}} \left(D^2\bar{D}^2 + \bar{D}^2 D^2\right) \frac{V}{\sqrt{\tilde{\alpha}}} + \\
&+ \int d^8z\; d^8z' \; 2i \;\; \mathrm{Tr}\left(b(z) + \bar{b}(z)\right) \frac{1}{\sqrt{\tilde{\alpha}(z)}} \; G^{(2)}_{[S,\bar{S},\sqrt{\tilde{\alpha}}]}(z-z') \times \\
&\times \delta_{\bar{c},c}\left(V \star {}^{(-1)}G^{(2)}_{[S,\bar{S},\sqrt{\tilde{\alpha}}]}\right)(z') + \int d^8z \; 2i\,\mathrm{Tr}\left(K \star G^{(2)}_{[S,\bar{S},\sqrt{\tilde{\alpha}}]}\right)(z) \times \\
&\times \delta_{\bar{c},c}\left(V \star {}^{(-1)}G^{(2)}_{[S,\bar{S},\sqrt{\tilde{\alpha}}]}\right)(z) + \int d^4y d^2\theta \; 2\,\mathrm{Tr}\;\; Lc^2 + \int d^4\bar{y} d^2\bar{\theta} \; 2\,\mathrm{Tr}\;\; \bar{L}\bar{c}^2 .
\end{aligned} \tag{15}$$

As usual, forth and fifth terms in the modified ST identities together with the ghost equation do not allow any correction to the gauge fixing term [9].

All the derivations proposed in this paper have a sense only if we have fixed a gauge invariant regularization and defined a renormalization scheme to remove infinities. In this work we implied the DRED scheme [11] that is the only practical regulator in order to be able to calculate higher order effects in any supersymmetric theory including MSSM. In this case ST identities (12) do not forbid a new gauge invariant term [12] of the effective action $\tilde{\Gamma}$ (14)

$$\int d^8 z \; g^{(\epsilon)}_{mn} \; \mathrm{Tr}\, \Gamma_m \Gamma_n \,,$$

where $g^{(\epsilon)}_{mn}$ is the metric in the 2ϵ compactified dimensions and Γ_m is the superfield gauge connection defined by

$$\Gamma_m = \frac{1}{2} \sigma_m{}^{\alpha\dot{\beta}} \bar{D}_{\dot{\beta}} \left(e^{-\tilde{V}} D_\alpha e^{\tilde{V}} \right).$$

This term generates so-called ϵ scalar masses [13] in the course of the renormalization procedure. Indeed, one can see that at the component level the $\theta^2\bar{\theta}^2$ component of $\tilde{z}_1$ that is the singular term of $G^{(2)}_{[S,\bar{S},\sqrt{\tilde{\alpha}}]}$ produces ϵ scalar masses when we are replacing $\tilde{V}$ with $V\star^{(-1)}G^{(2)}_{[S,\bar{S},\sqrt{\tilde{\alpha}}]}$ in $\tilde{\Gamma}$ to obtain the effective action Γ (15). Even if initially the ϵ scalar masses are equal to zero, this condition is unstable under renormalizations and, hence, such a counterterm must be added. As it has been found in [13] ϵ scalar masses dependence of the two-loop β functions can be completely removed by a slight modification of the DRED scheme to the $\overline{\mathrm{DR}}'$ scheme. The way how to generalize this scheme to all orders of the perturbation theory has been proposed in ref. [6]. However, based on the explicit presence of this contribution at the two-loop level in the component formalism [13], it has been stated in [14] that the contribution of the ϵ scalar mass renormalization should be taken into account in the physical soft scalar mass β functions. It is possible to determine this contribution at all orders of the perturbation theory by requiring the existence of a set of renormalization group invariant relations between soft couplings and masses as it has been done in ref. [14] for the $\overline{\mathrm{DR}}'$ scheme and further developed in refs. [15, 16] to other schemes. In ref. [17] authors explained appearance of the ϵ scalar contributions into the physical soft scalar mass β functions by using a non-trivial renormalization of the gauge fixing parameter $\tilde{\alpha}$ that is also a superfield and found a correspondence to the results obtained in refs. [14, 15, 16].

In summary, we have shown the solution to ST identities can be parametrized in the form (15). We have demonstrated that there are not quantum contributions into the effective action containing more than two ghost fields. This solution has been obtained without using the background field technique. By construction one can see that the solution is unique. This solution can have an application to calculations of process amplitudes in the MSSM containing components of gauge superfields as

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asymptotic states by using superfield formalism without going to components. Indeed, the approach proposed in our paper can be generalized to the case when matter superfields are present in the lagrangian, and a solution to ST identities can be found in that case too. At the same time, the Minimal Supersymmetric Standard Model as one of the supersymmetric theories with a spontaneously broken gauge symmetry through the Higgs mechanism can be described in terms of superfields in the broken phase by using the σ-model technique like it has been done in ref. [18]. The knowledge of the form (15) of the effective action can be used to simplify calculations of process amplitudes for the components of gauge supermultiplets of the MSSM since the gauge effective vertices of processes which can be read off from (15) are parametrized in terms of a 2-point ghost correlator ${}^{(-1)}G^{(2)}(z-z')$ only. In its turn, this correlator can be found in the perturbation theory, at least at the one-loop level. Calculations of this correlator are simpler than calculations of gauge effective vertices in a direct way within superfield formalism.

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